# Some Considerations on the Erosion and Deposition Rates of Sediment-water Mixture

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## INTRODUCTION

We investigated erosion/deposition rate of the sand-water flows by channel experiments, in which the velocities of both the clear water and mixture layer are measured by particle image velocimetry (PIV) technique (**Fig. 1**). Together with the help of Buckingham $\pi$-theorem, we figured out the factors of the erosion/deposition rate associated with the flow field over erodible bed. Based on experimental observation, a piecewise-linear distribution for velocity and a linear distribution of sediment concentration are speculated, as suggested by Itoh et al. (2005). With these assumptions, four $\pi$-terms ($\pi$-factors) related to the erosion/deposition processes are identified: a) the Bagnold number, in which the viscosity of the interstitial fluid is taken into account; b) the Savage number, in which the grain collision is considered; c) the ratio of the thickness of mixture layer to the total flow depth; d) the non-uniform velocity distribution along the flow thickness. The values of the parameters in the proposed formula for the erosion/ deposition rate are determined by means of regression with respect to experimental data in steady conditions (right top panel of **Fig. 1**). The resultant formula is then validated against experimental measurements for flows over a weir (right bottom panel of **Fig. 1)** and compared with other formulas in literature.

## FORMULATION, VALIDATION AND COMPARISIONS

We suppose that the erosion/deposition rate $E$ can be transformed by a dimensionless $\pi$-term, i.e. $\pi_1 = E\big/\left(u_m^2\, d_s/h_s\right)^{1/2} = E/\bar{u}_s$ , for figuring out a relation $\pi_1 = f(\pi_2,\pi_3,\pi_4,\pi_5)$ with $\pi_2 = N_{\text{Bag}}^{-1} = 2\mu_f h_s\big/\left(c_b \rho_s u_m d_s^2\right)$, $\pi_3 = N_{\text{Sav}}^{-1} = \Delta\rho g h_s^2 \cos\theta\big/\left(\rho_s u_m^2 d_s\right)$, $\pi_4 = c_b h_s/h$ , and $\pi_5 = \bar{u}_w/u_w$ . In this formulation, $N_{\text{bag}}$ is the Bagnold number, $N_{\text{Sav}}$ is the Savage number, $c_b$ means the sediment concentration just above the erodible bed, $\mu_f$ stands for the viscosity of the interstitial fluid, $h_s$ is the thickness of the mixture layer, $h$ is the total depth of the flow, $d_s$ is the representative

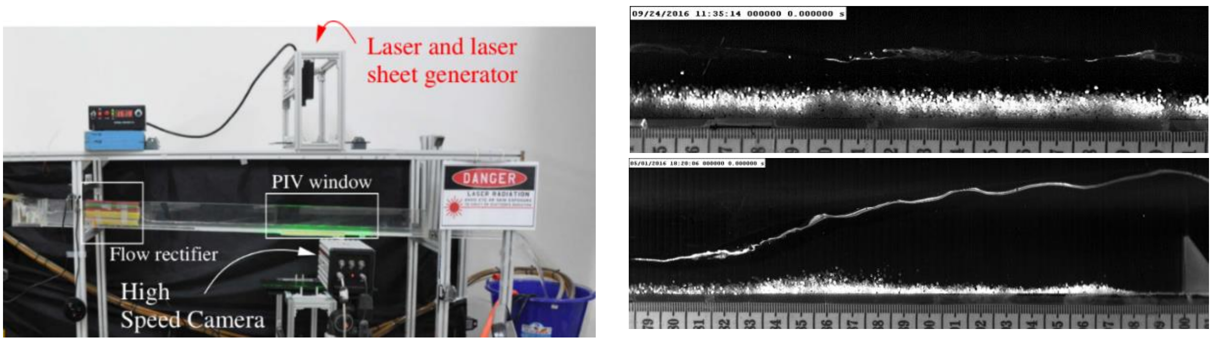


**Fig. 1** Experimental set-up (left panel); Determination of parameters: regression with respect to experimental data in steady conditions (right top panel); Validation: flows over a weir (right bottom panel).

concentration just above the erodible bed, $\mu_f$ stands for the viscosity of the interstitial fluid, $h_s$ is the thickness of the mixture layer, $h$ is the total depth of the flow, $d_s$ is the representative diameter of the sediment, $\theta$ is the local slope angle, $\Delta\rho=\rho_s-\rho_w$ is the difference between sediment and water, $\bar{u}_w$ is the depth-averaged flow velocity and $u_m$ is the flow velocity at the interface between the clear water layer and mixture layer. In addition, due to the density difference, the settlement process is supposed to exist in both erosion and deposition processes, i.e. $E=E_{ED}+S_s$ with $S_s$ the settling velocity. With respect to experimental data in steady states of erosion and deposition, the rate $E$ is determined by regression with respect to the form

$$E=E_{ED}+S_s,\qquad E_{ED}=\alpha_1 c_b \bar{u}_s A_\tau \pi_2^{\alpha_2}\pi_3^{\alpha_3}\pi_4^{\alpha_4}\pi_5^{\alpha_5}\quad\text{and}\quad S_s=\alpha_s\beta_D\omega_D\bar{c}\,\pi_2^{\beta_2}\pi_3^{\beta_3}\pi_4^{\beta_4}\pi_5^{\beta_5}. \tag{1}$$

In $(1)_2$, $A_\tau=(\tau_{sh}-\tau_e)/\tau_e$ is introduced to distinguish erosion and deposition, where

$$\tau_{sh}=\left(0.5u_m d_s h_s^{-1}\right)^2\left(gsd_s\cos\theta\right)^{-1},\ \ \tau_e=\left(s\bar{c}\tan\phi_s\right)\left(s\bar{c}+1\right)^{-1},\ \text{and}\ \ s=\Delta\rho/\rho_w\,. \tag{2}$$

In $(1)_3$, $\beta_D=\min\left(2,\ \left(1-c_\varphi\right)/\bar{c}\right)$ and $\omega_D=\left(A_d^2+1.09gsd_s\right)^{1/2}-A_d$ with $A_d=13.95\mu_f\rho_s^{-1}d_s^{-1}$, where $\bar{c}$ is the mean sediment concentration and $c_\varphi$ is the porosity of the sediment, cf. Li and Duffy (2011). All the parameters for $E_{ED}$ and $S_s$ are listed in **Table 1**. The left panel of **Figure 2** illustrates the prediction against experimental measurement at $\theta=5°$ in deposition case, where the sections (A, B, C, D, E) are depicted in the right panel and the foot notes (I, II, III, IV) indicate the time point for validation.

**Tab. 1** Parameters for $E_{ED}$ and $S_s$

| $E_{ED}$ | $\alpha_1$ | $\alpha_2$ | $\alpha_3$ | $\alpha_4$ | $\alpha_5$ |
|---|---|---|---|---|---|
| $\tau_{sh}>\tau_e$: Erosion | 1.04 | 0.3 | 0.7 | 2.6 | -1.3 |
| $\tau_{sh}<\tau_e$: Deposition | 461.13 | 1.0 | -0.9 | 7.1 | 0.2 |
| $S_s$ | $\alpha_s$ | $\beta_2$ | $\beta_3$ | $\beta_4$ | $\beta_5$ |
| | -0.58 | 0.1 | 0.1 | 2.0 | -1.1 |

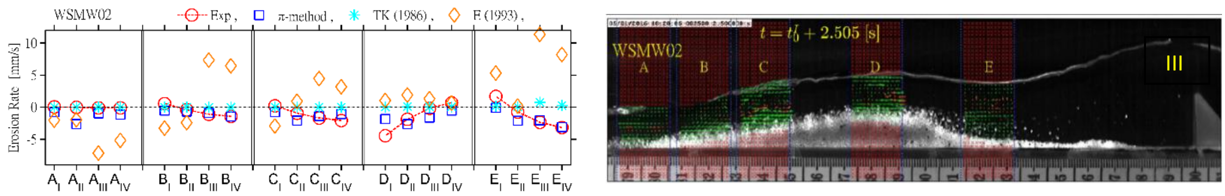


**Fig. 2 Left:** Comparison with respect to the experimental measurement, where TK (1986) means Takahashi and Kuang (1986), and E (1993) is for Egashira (1993); **Right:** Illustration of the locations of measurement section.

## CONCLUSIONS

By means of Buckingham π-theorem, a formulation for the erosion/ deposition rate is proposed, in which the viscosity of interstitial fluid (Bagnold number) and the characteristics of granular flows (Savage number) are included. The values of the parameters are determined by regression with respect to experimental data at steady state. The resultant formula is then validated against experimental measurements in different conditions and compared with other formulas in literature, e.g. Takahashi and Kuang (1986), Egashira (1993), etc.

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